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$N = 2$ supersymmetric FRW quantum cosmology from a D-*p*-brane gas*

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Abstract

In this paper, we investigate phenomenologically the possible effects of D-*p*-brane solitons within a quantum cosmological scenario, namely focusing on a $N = 2$ supersymmetric perspective. For this purpose, we consider a flat FRW model subject to a specific action for gravity and the dilaton. In particular, the coupling coefficient ω is now a function of $d = p + 1$, the dimension of the world volume swept by the *p*-brane in the physical space–time. Other fields (such as *n*-form fields associated with the D-*p*-brane) are represented by a perfect fluid satisfying consistent physical conditions. Subsequently, we find the general form of the corresponding quantum states (wave function of the universe), identifying a subset of these solutions that satisfy the requirements of $N = 2$ supersymmetry. These solutions also satisfy the physical properties of a duality transformation present in the effective action.

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1. Introduction

Recent developments in superstring theory suggest that, in the Planck-length regime, the quantum fluctuations are very large so that the coupling may increase and consequently the string degrees of freedom would not be the relevant ones (see e.g. discussions in [1–7] about such issues). Instead, solitonic degrees of freedom such as D-*p*-branes would become more important as the strong coupling regime becomes dominant. In particular, the quantum fluctuations will be strongly influenced by the effects of the D-*p*-branes. Hence, what would

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be the effect of those new physical degrees of freedom on, say, the very early universe and in particular from a quantum cosmological point of view?

In order to address this question, it is pertinent to stress the following. Quantum cosmology applies the fundamental principles of quantum physics to models of the very early universe. (For both a generic and thorough description and explanation on the many issues of quantum cosmology, see e.g. [8].) The vast majority of research in this field has focused on the family of spatially homogeneous cosmologies [9], where the configuration space is finite-dimensional. The Wheeler–DeWitt equation (the operatorial representation of the Hamiltonian constraint), corresponding to the gravitational version of the zero-energy Schrödinger equation, determines the evolution of the wavefunction of the universe [10]. A given trajectory mapped out by the wavefunction may be interpreted as a cosmological space–time[3]. Although the analysis of a finite number of degrees of freedom is restrictive, the expectation is that the main features of the wavefunction should be preserved in a more general analysis. However, such a framework does not admit a conserved current with a positive-definite probability density.

One possible resolution of this and other related difficulties in quantum cosmology is to extend the standard quantization of the universe in a *supersymmetric* fashion. (For relevant reviews based on different methods, see e.g. [12–17].) In recent years, two attractive (and possibly related) approaches have been pursued. One approach is to begin with $N = 1$ supergravity [18], employing a Hamiltonian formulation [19], in four dimensions and reduce the system to a one-dimensional model by invoking a suitable homogeneous ansatz [12–17, 20, 21]. This leads to a configuration space with $N = 4$ local supersymmetry. Alternatively, one may consider a purely bosonic one-dimensional model invariant under a global symmetry transformation. In this latter case, an $N = 2$ supersymmetry is induced within the configuration space if certain conditions are satisfied [22–31].

The physical framework just indicated is designated as supersymmetric quantum cosmology (SQC). Either $N = 2$ or $N = 4$ SQC is related to $N = 2$ or $N = 4$ supersymmetric quantum mechanics (SQM) (see the references in [32] for a detailed presentation of the formalism and physics of SQM), from which quite a few insights and techniques have been imported (see section 2). SQC constitutes an interesting and rewarding research topic[4] [12–17], providing the opportunity, on the one hand, to perform calculations that may be relevant for phenomenology and, on the other, having a close connection to exciting new areas of fundamental research such as quantum gravity, M/string theory and theoretical high energy physics in general.

The programme of research in SQC imports some of its guidelines from supergravity [18] using canonical methods [19]. It has been gradually enlarged, with many cosmological scenarios extensively reported in the published literature [12–17, 20–29, 31, 33–35]. The following properties enhace a significant motivation to conduct investigation in SQC:

- Firstly, research in SQC subscribes to the idea that treating both quantum gravity and supersymmetry effects as dominant will bring forward an improved description of the very early universe. This contrasts with conventional quantum cosmology [8], where quantum

[3] The retrieval of a classical space–time from within the framework of a quantum universe constitutes a pertinent problem in quantum cosmology [8]: our currently observed universe has a classical nature but it may have had a quantum origin. Its study has received quite wide attention, namely in the context of *decoherence* [11]. It proposes and explains how the universe could evolve from a quantum mechanical state, described, e.g., by a wave function, towards a situation in terms of classical equations of motion.

[4] The currently observed universe is neither quantum mechanical (see previous footnote) nor supersymmetric. Regarding this latter aspect, it is necessary that supersymmetry is broken somehow [18] in the evolution from a quantum-mechanical state towards a classical state. This is an important issue in SQC and some interesting recent contributions can be found in [33, 34].

gravity is present but *not* supersymmetry. In the SQC framework, we will therefore find a larger set of variables (bosonic and fermionic) as well as additional symmetries which increase the number of constraints, subsequently imposing a wider algebra.

- Moreover, $N = 1$ supergravity constitutes a natural 'square-root' of gravity in a Dirac-like manner [19]: the analysis of a second-order equation of the Klein–Gordon type (i.e., the Wheeler–DeWitt equation) could be substituted by that of a supersymmetry induced set of first-order differential equations. This would then have profound consequences regarding the dynamics of the wave function of the universe[5].
- To be more specific, in a FRW model the canonical representation is retrieved from the Hamiltonian formalism. We then find the Hamiltonian constraint, together with the supersymmetry constraints. These constraints should be satisfied by the physical states. From the form of the constraint algebra it may be sufficient that in particular models the supersymmetry constraints are sufficient to be solved.

Within the physical context of SQC (which constitutes a variant of SQM) described previously, we will apply it to obtain a set of quantum states associated with a specific FRW cosmological model. This model is implemented through the following result, introduced and discussed throughout [1–5]: within a p-brane cosmology scenario, at low energy, the effective action from the β-function of the string world sheet is not valid. Then, in order to obtain an adequate low energy effective theory with gravity and solitons, several frameworks have been proposed. Herewith we will employ the *phenomenological* approach, based on the useful result shown by Duff, Khuri and Lu [5] and further investigated in [1–4]: the natural metric that couples to a p-brane is the Einstein metric multiplied by a certain power of the dilaton field, inducing a particular Brans–Dicke action.

In this scenario, a very specific coupling $\omega(d)$ is obtained, depending on d, which is the corresponding dimension of the world-volume swept by the p-brane solution in the physical space–time. This overall setting represents a gas of solitonic p-branes [1–5], including a perfect fluid matter forming fields associated with the branes and originating from the appropriate compactification of the 10-dimensional low-energy effective theory. This determines an *arbitrary* relation $P = \gamma\rho$ for the pressure and energy densities in this FRW background (cf [1–5] for more details). This matter content can either be present in a Ramond/Ramond (R/R) or in a Neveu–Schwarz/Neveu–Schwarz (NS/NS) sector. In the former, the perfect fluid content is not coupled to the dilaton, while in the latter it is.

Accordingly, the purpose of this paper is to present an investigation on D-p-brane-induced quantum cosmology applied to a FRW geometry[6], extending it towards a $N = 2$ supersymmetric description. In order to contextualize the corresponding physical setting of this research work, a brief description on how $N = 2$ supersymmetry can be induced in some quantum cosmologies is presented in section 2. Employing then the above-mentioned modified action with a deformation parameter ω (now d-dependent [1–4]), we obtain in section 3 different quantum states. In section 4 we then show that some of the quantum states satisfy, the criteria

[5] It is important to remark the following. It has been shown (see [36] and other details in [12]) that the only non-trivial admissible quantum mechanical solution, satisfying the complete set of constraints of the full theory of $N = 1$ supergravity, is constituted by an infinite number of fermionic modes. No solution (besides the trivial one, $\Psi = 0$) can be found for states with a finite number of fermionic modes, in particular for a pure bosonic sector. This issue has also been discussed in [14–16]. An explicit solution consistent with this framework was presented in [37]. However, when one considers minisuperspace models (i.e., configuration spaces that are finite dimensional), then the set of constraints is less restrictive: a wide range of solutions with a bosonic sector of finite number of fermionic modes has then been found [12–17, 20–29, 31, 33–35].

[6] A generic and thorough analysis of these FRW quantum cosmologies, extracted in this D-p-brane phenomenological setting, will be presented elsewhere [38].

of $N = 2$ supersymmetry[7] [22–31]. Finally, we conclude the paper in section 5 with a discussion and outlook.

2. $N = 2$ SQC

Let us describe briefly how quantum cosmology relates to the usual quantum mechanics and therefore how SQC can be retrieved, relating it to an SQM context.

Suppose [8, 10] we are presented with a classical theory involving $n + 2$ variables $q^\alpha(t)$, $\alpha = 0, 1, \ldots, n$ and ℓ, described by an action

$$S[q^a, \ell] = \int \mathrm{d}t\, [(\ell - \tfrac{1}{2}\ell^2 \dot{q}^0)\delta_{ab}\dot{q}^a\dot{q}^b - W(q^a)\dot{q}^0], \tag{1}$$

where $a, b = 1, \ldots, n$ and a dot denotes differentiation with respect to time. This theory has an important symmetry. If for arbitrary f (but equal to unity at the endpoints of the integration interval) we make the transformation

$$\ell \rightarrow \dot{f}(t)\ell(f(t)); \qquad q^\alpha(t) \rightarrow q^\alpha(f(t)), \tag{2}$$

the action remains unchanged. One can easily see this by simultaneously changing the variable of integration $t \rightarrow f(t)$. For this reason this transformation is called a reparametrization of time. The classical equations of motion are found by varying with respect to ℓ, q^a, q^0. It is then found that the theory has a constraint: we are not free to specify all the ℓ, q^a, q^0 and their derivatives on some initial surface and integrate forward in time. Moreover, there is not even an evolution equation for ℓ. Only the q^a are the true degrees of freedom. The total Hamiltonian H is then found to satisfy

$$H = 0. \tag{3}$$

Quantum cosmology [8] is then, basically, quantum mechanics of a class of such systems with time-dependent configuration variables, satisfying the property of time-reparametrization invariance: $t \rightarrow t'(t)$ (see also [10]). This implies that the system is characterized by the Hamiltonian constraint (3). In more detail, the framework of quantum cosmology involves a class of models, where the field equations can be expressed in the form of an Hamiltonian system, where the Hamiltonian vanishes. The classical Hamiltonian constraint may be written in the more compact form

$$H = G^{ab}\pi_a\pi_b + W(q^a), \tag{4}$$

where q^a $(a = 1, 2, \ldots)$ are the configuration space variables, $G^{ab} = \mathrm{diag}(-1, 1, 1, \ldots)$ is the configuration space metric and $W(q^a)$ is the configuration space potential [8, 10]. By identifying the conjugate momenta with the operators $\pi_{q^a} = \pi_a = -\mathrm{i}\partial/\partial q^a$ and neglecting ambiguities that arise in the factor ordering [8] of some variables, we arrive at the Wheeler–DeWitt equation. The quantum states (wave function of the universe) are obtained by solving this equation

$$\hat{H}\Psi = 0, \tag{5}$$

where $\hat{H}$ is the Hamiltonian constraint operator. It is important to emphasize that due to the presence of G^{ab} in the 'kinetic' term $G^{ab}\pi_a\pi_b$, the Wheeler–DeWitt equation is of a hyperbolic

[7] In the context of supersymmetric cosmological models with perfect fluids, the interested reader will find relevant material in [26–28]. The important issue of factor ordering is addressed in [26, 27, 29], following a proposal for isospectral scheme introduced in [30].

nature (like the Klein–Gordon equation) and not in a direct correspondence to a Schrödinger-like form.

The usual SQM is obtained from extending standard quantum physics, as thoroughly explained throughout [32]. As far as SQC is concerned, how it can be retrieved has been pointed in section 1. As a careful analysis shows, SQC with $N = 2$ supersymmetry is a variant of $N = 2$ SQM with the added property of time-reparametrization invariance. This will determine, as explained above, that the total Hamiltonian will constitute a constraint and is equal to zero. Moreover, due to the presence of a $G^{ab}\pi_a\pi_b$ term, the corresponding differential equation is of an hyperbolic nature, i.e., similar to the Klein–Gordon equation. This implies, therefore, important differences that contrast with the usual SQM as described in [32], in spite of relevant similarities (either physical or merely formal) with SQC.

The procedure for attaining a $N = 2$ supersymmetric extension of some cosmological models can be summarized as follows. If the Euclidean Hamilton–Jacobi equation

$$G^{ab}\frac{\partial I}{\partial q^a}\frac{\partial I}{\partial q^b} = W(q^a) \tag{6}$$

admits a solution, $I = I(q^a)$, that respects the symmetry of the classical Hamiltonian [22–28, 31], a supersymmetric extension of the system is possible. In this case, a quantum Hamiltonian, $\hat{H}$, may be defined by the conditions

$$2\hat{H} = [Q, \bar{Q}]_+, \qquad Q^2 = \bar{Q}^2 = 0 \tag{7}$$

and

$$[\hat{H}, Q]_- = [\hat{H}, \bar{Q}]_- = 0, \tag{8}$$

where Q is a non-Hermitian supercharge and $\bar{Q}$ is its adjoint. The functional forms of these supercharges are

$$Q = \psi^a\left(\pi_a + \mathrm{i}\frac{\partial I}{\partial q^a}\right) \tag{9}$$

and

$$\bar{Q} = \bar{\psi}^a\left(\pi_a - \mathrm{i}\frac{\partial I}{\partial q^a}\right), \tag{10}$$

respectively, where the corresponding fermionic (Grassmannian) variables are defined by

$$\bar{\psi}^a = \theta^a, \qquad \psi^b = G^{ab}\frac{\partial}{\partial\theta^a}, \tag{11}$$

$$\psi^a\psi^b + \psi^b\psi^a = 0, \qquad \bar{\psi}^a\bar{\psi}^b + \bar{\psi}^b\bar{\psi}^a = 0. \tag{12}$$

Equations (7) and (8) represent the algebra for a $N = 2$ supersymmetry. In more precise terms, the classical Hamiltonian is viewed at the quantum level as the bosonic component of an $N = 2$ supersymmetric Hamiltonian. The quantum Hamiltonian subsequently has the form

$$H = H_0 + \frac{\hbar}{2}\frac{\partial^2 I}{\partial q^a\partial q^b}[\theta^a, \bar{\theta}^b]_+. \tag{13}$$

The supersymmetric wavefunction is then annihilated by the supercharges:

$$Q\Psi = 0, \tag{14}$$

$$\bar{Q}\Psi = 0, \tag{15}$$

and automatically satisfies the Hamiltonian constraint due to equation (7). Thus, the problem of quantizing these cosmological models in a $N = 2$ supersymmetric fashion involves finding a solution to Hamilton–Jacobi function that respects the global symmetries of the classical action and then solving both the supersymmetry constraints.

3. FRW quantum cosmology from D-*p*-branes

We take the following actions to describe FRW minisuperspaces extracted from a D-*p*-brane framework, employing the framework introduced and explained throughout [1–5], where the interested reader will find a detailed mathematical and physical description. For the R/R case we employ

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\{\mathrm{e}^{-\phi}[R - \omega(\nabla\phi)^2] + L_m\} \tag{16}$$

and for the NS/NS case we use

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\{\mathrm{e}^{-\phi}[R - \omega(\nabla\phi)^2 + L_m]\}, \tag{17}$$

where

$$\omega = -\frac{4(p-1)-(p+1)^2}{2(p-1)-(p+1)^2}, \tag{18}$$

for $p = d-1$. Herewith, d is the dimension of the world-volume swept by the extended object (e.g., a particle is a 0-brane, for $d = 1$ and $p = 0$, whereas a string is a 1-brane for $d = 2$ and hence $p = 1$). More generally, in a D-dimensional space time (in equation (18) we restrict ourselves henceforth to $D = 4$) we have

$$\omega = -\frac{(D-1)(n-3)-(n-1)^2}{(D-2)(n-3)-(n-1)^2}, \tag{19}$$

where $n = p + 2 = d + 1$ is the rank of the n-form field strength tensor associated with the p-brane object [5]. We further assume a flat FRW geometry with a metric given by the expression

$$\mathrm{d}s^2 = -N^2\,\mathrm{d}t^2 + \mathrm{e}^{2\alpha}\,\mathrm{d}\vec{x}^2 \tag{20}$$

and a perfect fluid matter content for L_m, satisfying $\rho = \rho_0\mathrm{e}^{-3(1+\gamma)\alpha}$. The reduced actions are therefore (see [1–3] for a detailed retrieval of these expressions) written (with overdot representing $\mathrm{d}/\mathrm{d}t$) as

$$S^{R/R} = \int \mathrm{d}t\,\mathrm{e}^{3\alpha-\phi}\left[\frac{1}{N^2}\{-6\dot{\alpha}^2 + 6\dot{\alpha}\dot{\phi} + \omega\dot{\phi}^2\} - N^2\rho_0\mathrm{e}^{-3(1+\gamma)\alpha+\phi}\right], \tag{21}$$

and

$$S^{NS/NS} = \int \mathrm{d}t\,\mathrm{e}^{3\alpha-\phi}\left[\frac{1}{N^2}\left\{-6\dot{\alpha}^2 + 6\dot{\alpha}\dot{\phi} + \omega\dot{\phi}^2\right\} - N^2\rho_0\mathrm{e}^{-3(1+\gamma)\alpha}\right]. \tag{22}$$

In addition, we will employ ([1–4, 39, 40]) a new time variable $\mathrm{d}t = \mathrm{d}\tau\mathrm{e}^{3\alpha-\phi}$ together with the parameters and variables $X(a,\phi)$, $Y(a,\phi)$ for the R/R case (see [1–4] for a thorough description of its use)

$$\kappa_{R/R} = 3(1-\gamma)^2(\omega - \omega_{\kappa_{R/R}}), \tag{23}$$

$$\mu_{R/R} = -\frac{8}{\kappa_{R/R}}(\omega - \omega_{\mu_{R/R}}), \tag{24}$$

$$\nu_{R/R} = -2(1-\gamma)(\omega - \omega_{\nu_{R/R}}), \tag{25}$$

$$\omega_{\kappa_{R/R}} = -\frac{4 - 6\gamma}{3(1-\gamma)^2}, \tag{26}$$

$$\omega_{\mu_{R/R}} = -\tfrac{3}{2}, \tag{27}$$

$$\omega_{\nu_{R/R}} = -\frac{1}{1-\gamma}, \tag{28}$$

$$-2X = 3(1-\gamma)\alpha - \phi, \tag{29}$$

$$Y = \alpha + \frac{\nu_{R/R}}{\kappa_{R/R}} X, \tag{30}$$

and for the NS/NS case

$$\kappa_{NS} = \tfrac{9}{4}(1-\gamma)^2(\omega - \omega_{\kappa_{NS}}), \tag{31}$$

$$\mu_{NS} = -\frac{3(2\omega + 3)}{\kappa_{NS}}, \tag{32}$$

$$\nu_{NS} = 3(1-\gamma)(\omega - \omega_{\nu_{NS}}), \tag{33}$$

$$\omega_{\kappa_{NS}} = \frac{4}{3}\frac{3\gamma - 1}{(1-\gamma)^2}, \tag{34}$$

$$\omega_{\nu_{NS}} = -\frac{2}{1-\gamma}, \tag{35}$$

$$-2X = 3(1-\gamma)\alpha - 2\phi, \tag{36}$$

$$Y = \alpha + \frac{\nu_{NS}}{2\kappa_{NS}} X. \tag{37}$$

From these we then write the actions (21), (22) in the same formal manner, as the following reduced actions[8] (where prime denotes $\mathrm{d}/\mathrm{d}\tau$):

$$S^{R/R} = \int \mathrm{d}\tau \left[\frac{1}{N^2}\{3\kappa_{R/R}Y'^2 + \mu_{R/R}X'^2\} - N^2\rho\mathrm{e}^{-2X}\right], \tag{38}$$

and

$$S^{NS} = \int \mathrm{d}\tau \left[\frac{1}{N^2}\{3\kappa_{NS}Y'^2 + \mu_{NS}X'^2\} - N^2\rho\mathrm{e}^{-2X}\right]. \tag{39}$$

Let us now extend the cosmological framework presented in [1–4], introducing herein a corresponding canonical Hamiltonian formulation. After a suitable variable redefinition (up to constant factors), we obtain

$$\epsilon_{\kappa_{R/R}}\pi_Y^2 + \epsilon_{\mu_{R/R}}\pi_X^2 + 4\rho_0\mathrm{e}^{-2X/m_{R/R}^{1/2}} = 0, \tag{40}$$

[8] See [1–4] for additional details about these computational steps.

and

$$\epsilon_{\kappa_{NS}}\pi_Y^2 + \epsilon_{\mu_{NS}}\pi_X^2 + 4\rho_0 e^{-2X/m_{NS}^{1/2}} = 0. \tag{41}$$

In obtaining the above Hamiltonian expressions we used the following procedure. For the R/R case, e.g., we re-write $3\kappa_{R/R}Y'^2 \equiv \overline{Y'}^2$ and put $\kappa_{R/R} = |\kappa_{R/R}|\epsilon_{\kappa_{R/R}} \equiv k_{R/R}\epsilon_{\kappa_{R/R}}$, where $k_{R/R} > 0$ and $\epsilon_{\kappa_{R/R}} \equiv \operatorname{sign}\kappa_{R/R}$. Then we drop the 'hat-bar' over all the variables. Accordingly, we have then used the following notations:

$$\begin{aligned}
\kappa_{R/R} &= k_{R/R}\epsilon_{\kappa_{R/R}}, & \epsilon_{\kappa_{R/R}} &\equiv \operatorname{sign}\kappa_{R/R} = \pm 1,\\
\kappa_{NS} &= k_{NS}\epsilon_{\kappa_{NS}}, & \epsilon_{\kappa_{NS}} &\equiv \operatorname{sign}\kappa_{NS} = \pm 1,\\
\mu_{R/R} &= m_{R/R}\epsilon_{\mu_{R/R}}, & \epsilon_{\mu_{R/R}} &\equiv \operatorname{sign}\mu_{R/R} = \pm 1,\\
\mu_{NS} &= m_{NS}\epsilon_{\mu_{NS}}, & \epsilon_{\mu_{NS}} &\equiv \operatorname{sign}\mu_{NS} = \pm 1.
\end{aligned} \tag{42}$$

The corresponding Wheeler–DeWitt equations are thus

$$-\epsilon_{\kappa_{R/R}}\frac{\partial^2\Psi_{R/R}}{\partial Y^2} - \epsilon_{\mu_{R/R}}\frac{\partial^2\Psi_{R/R}}{\partial X^2} + 4\rho_0\, e^{-2X/m_{R/R}^{1/2}}\Psi_{R/R} = 0, \tag{43}$$

and

$$-\epsilon_{\kappa_{NS}}\frac{\partial^2\Psi_{NS}}{\partial Y^2} - \epsilon_{\mu_{NS}}\frac{\partial^2\Psi_{NS}}{\partial X^2} + 4\rho_0\, e^{-2X/m_{NS}^{1/2}}\Psi_{NS} = 0. \tag{44}$$

Let us address the issue of solving the above equations (43), (44). As far as the most general spectrum of states is concerned, one important aspect to take into consideration is to determine the regions in the (γ, ω) space where $\kappa_{R/R}$, κ_{NS}, $\mu_{R/R}$, and μ_{NS} are positive, zero or negative, i.e., where $\epsilon_{\kappa_{R/R}}, \epsilon_{\kappa_{NS}}, \epsilon_{\mu_{R/R}}$and $\epsilon_{\mu_{NS}}$ are $+1$, -1 or zero. This will be of relevance when we take $\rho_0 > 0$ or $\rho_0 < 0$. In order to solve the Wheeler–DeWitt equations, we then re-write both in a common formal expression as

$$\left[-s_1\frac{\partial^2}{\partial y^2} - s_2\frac{\partial^2}{\partial x^2} + 4\rho_0 e^{-2x/M}\right]\Psi = 0, \tag{45}$$

where $s_1 \equiv \epsilon_{\kappa_{R/R}}$ or $\epsilon_{\kappa_{NS}}$, $s_2 \equiv \epsilon_{\mu_{R/R}}$ or $\epsilon_{\mu_{NS}}$ and $M \equiv \sqrt{m_{R/R}}$ or $\sqrt{m_{NS}}$, $y \equiv Y, x \equiv X$. Equation (45) is in a mathematical form that includes those previously used in many other publications dealing with quantum cosmological models retrieved from superstring theory (see e.g. [24, 25, 31, 39–42]). We follow very closely herein the procedures indicated and thoroughly described in those publications (see also [23, 26–29]). We take the following ansatz for the wave function

$$\Psi = \chi(x)\xi(y), \tag{46}$$

and the solutions will take the form

$$\xi(y) = \begin{cases} e^{\pm iEy} & \text{if } s_1 = 1,\\ e^{\pm Ey} & \text{if } s_1 = -1,\end{cases} \tag{47}$$

where E is a separation constant. Henceforth, we choose the physical sector where $\rho_0 > 0$ and the corresponding admissible values of s_2. Subsequently, the solutions physically interesting for $\chi(x)$ are as follows:

- If $s_2 = -1$, then following the framework and indications pointed out in [24, 25, 31, 39–42] and employing also [43–45], we have a linear combination of Bessel functions of the first and second kind of order $\pm iEM$ with argument $z = 2\sqrt{\rho_0}Me^{-x/M}$.

- If $s_2 = 1$, then we have a linear combination of Bessel functions of the first and second kind of order $\pm EM$ with argument $z = 2\mathrm{i}\sqrt{\rho_0}M\mathrm{e}^{-x/M}$.

4. N = 2 supersymmetry in a D-p-brane FRW quantum cosmology

Within models induced by string theory, an important feature is that some are characterized by a classical global symmetry that leaves the Lagrangian and Hamiltonian invariant. The four-dimensional Brans–Dicke theory is relevant to the early universe and arises as the effective action of higher-dimensional gravity theories, in particular 10-dimensional superstring theory (see [41] for a broader explanation). The spatially flat and isotropic Brans–Dicke cosmology exhibits then a discrete 'scale factor duality'. This symmetry forms the basis of the pre-big bang inflationary scenario [46, 47] and its origin can be traced to the T-duality of string theory. The consequences of scale factor duality for string quantum cosmology have been explored by a number of authors [46]. In particular, it has been pointed that the duality could be related to a 'hidden' supersymmetry [24, 25].

This is an important aspect to investigate within our above quantum cosmological model. In the variables X, Y, we have the following duality for the scenario induced by D-p-branes degrees of freedom

$$X = \overline{X}, \qquad Y = -\overline{Y}, \tag{48}$$

which translates as, in the (α, ϕ) coordinates, respectively for the R/R and NS case,

$$\overline{\alpha} = \left[-1 - 6(\gamma - 1)\frac{\nu_{R/R}}{2\kappa_{R/R}}\right]\alpha - \frac{\nu_{R/R}}{\kappa_{R/R}}\phi, \tag{49}$$

$$\overline{\phi} = 6(\gamma - 1)\left[1 + 3(\gamma - 1)\frac{\nu_{R/R}}{2\kappa_{R/R}}\right]\alpha - \left[1 + 3(\gamma - 1)\frac{\nu_{R/R}}{\kappa_{R/R}}\right]\phi, \tag{50}$$

and

$$\overline{\alpha} = -\left[1 + \frac{3}{2}(\gamma - 1)\frac{\nu_{NS}}{\kappa_{NS}}\right]\alpha - \frac{\nu_{NS}}{\kappa_{NS}}\phi, \tag{51}$$

$$\overline{\phi} = \frac{3}{2}(\gamma - 1)\left[2 + \frac{3}{2}(\gamma - 1)\frac{\nu_{NS}}{2\kappa_{NS}}\right]\alpha + \left[1 + \frac{3}{2}(\gamma - 1)\frac{\nu_{NS}}{\kappa_{NS}}\right]\phi, \tag{52}$$

with

$$\frac{\nu_{R/R}}{\kappa_{R/R}} = \frac{2}{3}\frac{\omega(1-\gamma) + 1}{\omega(1-\gamma)^2 + (4 - 6\gamma)/3}, \tag{53}$$

$$\frac{\nu_{NS}}{\kappa_{NS}} = \frac{\omega(1-\gamma) + 2}{\frac{3}{4}\omega(1-\gamma)^2 - (3\gamma - 1)}. \tag{54}$$

In the presence of this duality one may check for wave function solutions with $N = 2$ SUSY quantum cosmology (i.e., according to [22–25, 31]; see also [26–29, 33, 34]). Basically, one needs to consider from the Wheeler–DeWitt equation

$$\left[-s_1\frac{\partial^2}{\partial y^2} - s_2\frac{\partial^2}{\partial x^2} + 4\rho_o\mathrm{e}^{-2x/M}\right]\Psi = 0 \tag{55}$$

the corresponding Euclidean Hamilton–Jacobi equation

$$G^{ab} = \frac{\partial I}{\partial q^a}\frac{\partial I}{\partial q^b} = W(q^a), \tag{56}$$

with

$$G^{ab} \equiv (s_1, s_2); \qquad q^a = (y, x) = (q^0, q^1) \tag{57}$$

and

$$W = 4\rho_0 \mathrm{e}^{-2x/M} = W(x), \tag{58}$$

i.e.,

$$s_1 \left(\frac{\partial I}{\partial y}\right)^2 + s_2 \left(\frac{\partial I}{\partial x}\right)^2 = W(x) = 4\rho_0 \mathrm{e}^{-2x/M}. \tag{59}$$

Following an explanatory context similar to the previous section, let us also mention that equation (59) is formally such that it includes expressions previously used in previous papers addressing supersymmetric quantum cosmologies (see e.g. [24, 25, 31, 41]). Herein, we will employ very closely the procedures used and described in those references. We hence seek solutions to the above equation. One possibility is

$$I = J(x) + Ay. \tag{60}$$

Following the steps described in detail in [24, 25, 31], we find that it reduces to

$$s_2 \left(\frac{\mathrm{d}J}{\mathrm{d}x}\right)^2 = 4\rho_0 \mathrm{e}^{-2x/M} - s_1 A^2. \tag{61}$$

A general set of solutions for equation (61) (and hence equation (59)) will be determined by the choices of whether s_1 and/or s_2 are ± 1 or zero, together with choosing $\rho_0 > 0$ or $\rho_0 < 0$ (and so whether $W(x) = 4\rho_0 \mathrm{e}^{-2x/M}$ is positive, negative or zero). An analysis can be made according to four situations:

- Case 1: $s_2 = 1, s_1 = 1$;
- Case 2: $s_2 = -1, s_1 = 1$;
- Case 3: $s_2 = -1, s_1 = -1$;
- Case 4: $s_2 = 1, s_1 = -1$.

In each it can be investigated what the values/sign of ρ_0 and the term $4\rho_0 \mathrm{e}^{-2x/M} - s_1 A^2$ determine.

Nevertheless, we employ the same line as in the previous section and restrict ourselves to the sector $\rho_0 > 0$, thus considering the set of solutions with physical significance. The solutions for $J(x)$ do exist (see [24, 25, 31, 41] and [43–45]) and satisfy the duality properties (49)–(52) above. Thus, we can have a $N = 2$ supersymmetric extension for our model. Let us take the case $A = 0$ for simplicity where we obtain[9]

$$s_2 \left(\frac{\mathrm{d}J}{\mathrm{d}x}\right)^2 = 4\rho_0\, \mathrm{e}^{-2x/M}, \tag{62}$$

determining $s_2 = 1$ for $\rho_0 > 0$. The solutions are

$$J = \mp 2\sqrt{\rho_0} M\, \mathrm{e}^{-x/M}. \tag{63}$$

[9] Equation (62) includes others previously used (see e.g. [24, 25, 31, 41]) regarding the use of the Euclidean Hamilton–Jacobi equation in SQC. We follow the steps introduced therein.

Following the method outlined in references such as [22–29, 31] we can establish the $N = 2$ SUSY constraints as follows:

$$Q = \psi^a \left(\pi_a + \mathrm{i} \frac{\partial I}{\partial q^a} \right), \tag{64}$$

$$\overline{Q} = \overline{\psi}^a \left(\pi_a - \mathrm{i} \frac{\partial I}{\partial q^a} \right), \tag{65}$$

with $\overline{\psi}^a \equiv \theta^a$, $\psi^b \equiv G^{ab}\, \partial/\partial\theta^a$ being Grassmanian variables and satisfying the *ansatz* for the $N = 2$ SUSY wave function of the universe as (see e.g. [22–29, 31])

$$\Psi_{N=2} \equiv A_+(q^a) + B_a(q^a)\theta^a + A_-(q^a)\theta^1\theta^2, \tag{66}$$

with $B_a\theta^a = B_1\theta^1 + B_2\theta^2$. Now we apply

$$\overline{Q}\Psi_{N=2} = 0, \tag{67}$$

$$Q\Psi_{N=2} = 0, \tag{68}$$

to get a set of first-order partial differential equations, whose solutions include

$$A_+ \sim \mathrm{e}^{f}, \tag{69}$$

$$A_- \sim \mathrm{e}^{-f}, \tag{70}$$

$$f \equiv \pm 2\sqrt{\rho_0} M\, \mathrm{e}^{-x/M} = J. \tag{71}$$

Moreover, the terms B_1 and B_2 satisfy the Laplace equation (see [22–29, 31] for details or discussions on these components of the wave function).

5. Discussion and outlook

The possible effects of D-p-brane solitons within a quantum cosmological scenario were investigated from a phenomenological perspective and within a $N = 2$ supersymmetric point of view. The motivations were the framework and results outlined in [1–4]. As a consequence, we employed the flat FRW model introduced in those publications, which was described by a *specific* action for gravity and the dilaton. The details of the computational steps can be found in [1–4] (see also [39–42, 46, 47] for additional elements and physical context). In particular, the coupling coefficient ω was taken to be a function of $d = p + 1$, the dimension of the world volume swept by the p-brane in the physical space–time. In this paper, after obtaining the corresponding Hamiltonian, we retrieved the general form of the corresponding quantum physical states (wave function of the universe), identifying a subset of these solutions that satisfy the requirements of $N = 2$ supersymmetry.

Two pertinent issues regarding the approach and method employed herewith ought to be mentioned.

Firstly, we point out that it may not be justified to quantize an effective theory with actions (21) and (22) (arising from a fundamental quantum theory as superstring or M-theory). However, in so far as new fundamental fields and effects arising from the fundamental theory, a quantization of the effective action could capture significant and relevant novel features.

Secondly, it is interesting to compare the wavefunction for the empty fermion sector with the general solution to the bosonic Wheeler–DeWitt equation. A class of solutions of the latter include a linear combination

$$\Psi = c_1 I_0(f) + c_2 K_0(f), \tag{72}$$

where I_0 and K_0 are Bessel functions of the first and second kind respectively with order zero, f satisfies the Hamilton–Jacobi equation and c_i are arbitrary constants. In the large argument limit, the Bessel function of the first kind asymptotes to the form $I_0 \propto f^{-1/2}\exp(f)$ and, consequently, there is a correlation, up to a negligible prefactor, with the fully bosonic component A_+ of the supersymmetric wavefunction (see (66) and section 4).

Let us conclude this paper by raising an important aspect in the physical context of SQC (and therefore SQM). $N = 2$ induced SQC, extracted within superstring theories (bearing scale-factor dualities), holds pertinent similarities in comparison with configuration spaces retrieved from $N = 1$ quantum supergravity. However, there are delicate structural differences. Although both have a canonical structure with an Hamiltonian constraint set to zero (and inducing a bosonic quantum mechanical hyperbolic differential equation), the latter leads to $N = 4$ supersymmetric configuration spaces, but the former seems to induce $N = 2$ SUSY exclusively. A clear relation between these two descriptions in SQC (that constitute particular variants of SQM) is yet an open issue for future investigation, that could bring about novel insights concerning a quantum mechanical description of the very early universe in the presence of supersymmetry.

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